

IGCSE Additional Mathematics (0606) Master Revision Sheet

Aligned with CAIE Syllabus — Free Study Asset from <https://igcsegradecalc.site>

1. Functions & Inverse Functions

- **Composite Functions:** $(f \circ g)(x) = f(g(x))$. Note: $f(g(x)) \neq g(f(x))$ in general.
- **Inverse Functions:** $f^{-1}(x)$ exists **ONLY** if $f(x)$ is a one-to-one function.
- **Domain & Range:**
 - Domain of f^{-1} = Range of f
 - Range of f^{-1} = Domain of f
- **Graphical Property:** $y = f(x)$ and $y = f^{-1}(x)$ are reflections in the line $y = x$.

2. Quadratic Functions & Inequalities

- **Standard Form:** $y = ax^2 + bx + c$
- **Vertex Form:** $y = a(x - h)^2 + k \implies$ Vertex at (h, k)
- **Quadratic Formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **Discriminant** $D = b^2 - 4ac$:
 - $D > 0 \implies$ 2 distinct real roots (intersects x-axis twice)
 - $D = 0 \implies$ 1 real root / tangent to x-axis
 - $D < 0 \implies$ No real roots (completely above/below x-axis)
 - **Exam Tip:** Line $y = mx + c$ is tangent to curve $y = ax^2 + bx + c \iff b^2 - 4ac = 0$ after equating them.

3. Equations, Inequalities & Absolute Value

- **Modulus Equations:** $|f(x)| = k \implies f(x) = k$ or $f(x) = -k$
- **Modulus Graph:** To draw $y = |f(x)|$, sketch $y = f(x)$ and reflect the portion below the x-axis upward.
- **Simultaneous Equations:** Substitute linear into non-linear equation to form a quadratic in one variable.

4. Indices, Surds & Logarithms

- **Laws of Indices:**

$$a^m \times a^n = a^{m+n}, \quad a^m \div a^n = a^{m-n}, \quad (a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}, \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}, \quad a^0 = 1$$

- **Rationalising Surds:**

$$\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}, \quad \frac{1}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$$

- **Laws of Logarithms** (Base $a > 0, a \neq 1$):

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^k) = k \log_a x, \quad \log_a a = 1, \quad \log_a 1 = 0$$

- **Change of Base Formula:** $\log_a b = \frac{\log_c b}{\log_c a} = \frac{\ln b}{\ln a}$
- **Natural Logarithms:** $\ln x = \log_e x \implies e^{\ln x} = x, \quad \ln(e^x) = x$

5. Polynomials & Remainder/Factor Theorems

- **Remainder Theorem:** If polynomial $P(x)$ is divided by $(ax - b)$, the remainder is $R = P\left(\frac{b}{a}\right)$.
- **Factor Theorem:** $(ax - b)$ is a factor of $P(x) \iff P\left(\frac{b}{a}\right) = 0$.

6. Linear Law (Straight Line Graphs)

- Convert non-linear relationship $y = f(x)$ into $Y = mX + c$:
 - $y = ab^x \implies \lg y = (\lg b)x + \lg a$ ($Y = \lg y, X = x$)
 - $y = ax^b \implies \lg y = b \lg x + \lg a$ ($Y = \lg y, X = \lg x$)
 - $xy = a + bx \implies y = a\left(\frac{1}{x}\right) + b$ ($Y = y, X = \frac{1}{x}$)

7. Series: AP & GP

- **Arithmetic Progression (AP):**
 - n -th term: $T_n = a + (n - 1)d$
 - Sum of first n terms: $S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$
- **Geometric Progression (GP):**
 - n -th term: $T_n = ar^{n-1}$
 - Sum of first n terms: $S_n = \frac{a(1 - r^n)}{1 - r}$ ($r \neq 1$)
 - **Sum to Infinity:** $S_\infty = \frac{a}{1 - r}$ ONLY valid if $|r| < 1$

8. Circular Measure (Radians)

- **Conversion:** $\pi \text{ rad} = 180^\circ \implies 1 \text{ rad} = \frac{180^\circ}{\pi}$
- **Arc Length:** $s = r\theta$ (θ in radians)
- **Sector Area:** $A = \frac{1}{2}r^2\theta$ (θ in radians)
- **Segment Area:** $A = \frac{1}{2}r^2(\theta - \sin \theta)$

9. Trigonometry

- **Identities:**

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta, \quad 1 + \cot^2 \theta = \csc^2 \theta$$

- **Reciprocal Functions:**

$$\sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

- **Sine & Cosine Rules:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \implies \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\text{Area of Triangle} = \frac{1}{2} ab \sin C$$

10. Permutations & Combinations

- **Factorial:** $n! = n \times (n-1) \times \dots \times 1$, $0! = 1$
- **Permutations** (Order matters): ${}^n P_r = \frac{n!}{(n-r)!}$
- **Combinations** (Order does NOT matter): ${}^n C_r = \frac{n!}{r!(n-r)!}$

11. Differentiation (Calculus)

- **Basic Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- **Product Rule:** $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
- **Quotient Rule:** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
- **Trig Derivatives:**

$$\frac{d}{dx}(\sin ax) = a \cos ax, \quad \frac{d}{dx}(\cos ax) = -a \sin ax$$

$$\frac{d}{dx}(\tan ax) = a \sec^2 ax$$

- **Exponential & Log Derivatives:**

$$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}, \quad \frac{d}{dx}(\ln(ax+b)) = \frac{a}{ax+b}$$

- **Stationary Points & Nature:**

- At stationary point: $\frac{dy}{dx} = 0$
- $\frac{d^2y}{dx^2} > 0 \implies$ Minimum point
- $\frac{d^2y}{dx^2} < 0 \implies$ Maximum point

- **Connected Rates of Change:** $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

12. Integration (Calculus)

- **Indefinite Integrals:**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

- **Trigonometric Integrals:**

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$$

$$\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$$

- **Definite Integration & Area:**

- Area between curve and x-axis: $A = \int_a^b y dx$
- Area between curve and y-axis: $A = \int_c^d x dy$

13. Kinematics (Motion in Straight Line)

- Displacement s , Velocity v , Acceleration a :

$$v = \frac{ds}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$s = \int v dt, \quad v = \int a dt$$

- **Key Concepts:**

- Particle at instantaneous rest $\implies v = 0$
- Maximum velocity $\implies a = \frac{dv}{dt} = 0$
- **Total Distance** in time T : Calculate $\int |v| dt$ by splitting integral where $v = 0$.

14. Vectors in 2D

- **Position Vector:** $\mathbf{r} = x\mathbf{i} + y\mathbf{j} = \begin{pmatrix} x \\ y \end{pmatrix}$
- **Magnitude:** $|\mathbf{r}| = \sqrt{x^2 + y^2}$
- **Unit Vector in direction of $\mathbf{a}$:** $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$
- **Relative Position:** $\vec{AB} = \vec{OB} - \vec{OA} = \mathbf{b} - \mathbf{a}$
- **Parallel Vectors:** $\mathbf{a} \parallel \mathbf{b} \iff \mathbf{a} = k\mathbf{b}$ (where k is a scalar)